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Technical Comments

Comment on "Matrix Method for Eigenstructure Assignment: The Multi-Input Case with Application"

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Introduction

IN Ref. 1, a method for assigning eigenstructure to a linear time-invariant multi-input system is proposed. In the paper, the authors asserted that their method can assign the desired eigenvalues and eigenvectors at the desired locations. Based on the previous results on eigenstructure assignment, the authors proposed a novel computation method for a feedback gain matrix using some matrix operations in the subsection Computation of State Feedback Controller Gain. Another contribution of the paper is an application of their method to tethered satellite system (TSS). In the application, control of a space platform-based TSS is considered to assess the validity of their proposed algorithm.

In this Technical Comment, some crucial mistakes are pointed out. One is in the calculation of the feedback gain matrix F in Eq. (14) of the paper. Another one is the selection of the desired eigenvectors to decouple the platform from tether and offset dynamics in their application. In their formulation for computing the feedback gain matrix F [see Eq. (14) of Ref. 1], the pseudoinverse is used because the control input matrix B has rank deficiency in general. In other words, the gain F is obtained in the least square sense. This may cause inconsistency between the desired eigenvalues and the achieved eigenvalues. The primary objective of eigenstructure assignment is to assign the eigenvalues at the desired locations. The eigenvector assignment problem is an accompanying problem using the remaining freedom beyond closed-loop eigenvalue assignment. Thus, the exact achievement of the desired eigenvalues is very important requirement in eigenstructure assignment. In the sequel, the stability of the closed-loop system may not be guaranteed because the gain F is obtained in the least square sense. This is verified by a counterexample to follow. In addition, the selection of the desired eigenvectors in their application is not appropriate. The desired eigenvalues considered in the application are given in a complex-valued form. But the corresponding desired eigenvectors are given in a real-valued form. This fact does not agree with the previous result of Andry et al.[2] Thus, the desired eigenvectors should be changed to a complex-valued form corresponding to the complex-valued desired eigenvalues. A counterexample for illustrating the inconsistency mentioned is given in the following section.

Counterexample

Consider the following third-order controllable system:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Let the desired eigenvalues be −2, −3, −4; then the matrix A_d defined in Eq. (11b) of Ref. 1 can be represented by $A_d = \text{diag}\{-2, -3, -4\}$. The structure of the desired eigenvectors can be selected as

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then according to the procedure of the paper, the achievable eigenvectors are given as follows:

$$T = \begin{bmatrix} 0.2 & -0.3 & 0 \\ -0.4 & 0.9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the feedback gain matrix F is computed using the obtained T matrix and given matrices as follows:

$$F = \begin{bmatrix} 2 & -1 & -6 \\ 1.3333 & -5 & -1 \end{bmatrix}$$

Using the obtained gain matrix and, therefore, the eigenvalues of the resulting closed-loop system, $(A + BF)$ are given as 0.2538, −5.2538, and −4, which are considerably different from the desired ones (i.e., −2, −3, −4), and the closed-loop system is unstable because the feedback gain F is obtained in the least square sense. This verifies the inconsistency of the desired eigenvalues and the achieved ones.

References

[1] Pradhan, S., Modi, V. J., Bhat, M. S., and Misra, A. K., "Matrix Method for Eigenstructure Assignment: The Multi-Input Case with Application," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 5, 1994, pp. 983–989.

[2] Andry, A. N., Jr., Shapiro, E. Y., and Chung, J. C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-19, No. 5, 1983, pp. 711–729.

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Reply by the Authors to Choi et al.

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CHOI et al. point out two "crucial mistakes." The first one pertains to the feedback gain matrix F in Eq. (14). This is an

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outcome of a typographical error in the similarity relation where x and z are interchanged, i.e., the true relation should be $x = Tz$. This, in turn, affects Eq. (14), where the term $T^{-1}A_dT$ becomes TA_dT^{-1}. However, these in no way affect the algorithm and results presented in the paper. In fact, the modifications lead to the exact eigenvalue assignment in the "counterexample" considered by Choi et al.

As to the second comment, the state variables can be arranged arbitrarily, resulting in a corresponding modal matrix (V) to decouple the desired degrees of freedom. In the given example of a tether system, it seems logical that one may want to decouple the platform from the tether and offset dynamics. To that end, the structure of the desired eigenvectors can be selected appropriately. However, here V is taken arbitrarily. Now the platform and tether with offset dynamics are no longer decoupled, and the matrix V is so selected as to meet the closed-loop performance requirement, as mentioned in the paper. Obviously, if one attempts to decouple the platform from the tether and offset dynamics with this V, the state vector should be rearranged as $x = \{\alpha_p \;\; \dot{\alpha}_p \;\; \alpha_t \;\; d_{p_z} \;\; \dot{\alpha}_t \;\; \dot{d}_{p_z}\}^T$.

Concerning the choice of elements in the desired eigenvectors, neither is there a need to agree with the procedure of Andry et al., as Choi et al. suggest, nor we have attempted to do so. In fact, Andry et al. nowhere insist that the desired eigenvectors be complex. This of course is logical as long as chosen values, real or complex, lead to a desired or permissible response. In our study, it was achieved through the choice of real values for the desired eigenvectors. Note that, although the desired eigenvectors are taken to be real, the assignable eignevectors corresponding to the desired eigenvalues are complex.

Comments by Choi et al. are well appreciated.